INTERNATIONAL MATHEMATICS
TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 41, Northern Autumn 2019 (O Level)

Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. An illusionist lays a full deck of 52 cards in a row and tells spectators that 51 cards will be taken away step by step with only the Three of Clubs remaining on the table. On each step some spectator tells the illusionist a number so that a card lying on the place with this number in the row is taken away. However, the illusionist makes his own decision from which side of the row, left or right, he should count that number from to take the card away. For which initial positions of the Three of Clubs can the illusionist guarantee the success of his trick for sure? (4 points)

2. Let ω be a circle with the centre O and two different points A and C on ω. For any point P on ω distinct from A and C let X and Y be the midpoints of AP and CP respectively. Futhermore, let H be the point where the altitudes of triangle OXY meet. Prove that the position of the point H does not depend on the choice of P. (4 points)

3. There is a row of 100 squares each containing a counter. Any 2 neighbouring counters can be swapped for 1 dollar and any 2 counters that have exactly 3 counters between them can be swapped for free. What is the least amount of money that must be spent to rearrange the counters in reverse order? (4 points)

4. Let $a_1, \ldots, a_{1000}$ be given integers. Their squares $a_1^2, \ldots, a_{1000}^2$ are written around a circle. It is known that the sum of any 41 consecutive numbers on this circle is a multiple of 41^2. Is it necessarily true that each of the integers $a_1, \ldots, a_{1000}$ is a multiple of 41? (5 points)

5. Vasya has an unlimited supply of bricks of size $1 \times 1 \times 3$ and L-shape bricks, both made of three cubes of size $1 \times 1 \times 1$. Vasya completely filled a box of size $m \times n \times k$ with these bricks, where m, n and k are integers greater than 1. Prove that he can completely fill the same box using L-shape bricks only. (5 points)